Wire-rad: A teaching program for the study of radiation process in a center-fed linear antenna

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Abstract— **Este trabajo es una muestra de los objetivos desarrollados dentro de los Proyectos de Innovación Docente del Vicerrectorado de Planificación, Calidad y Evaluación Docente de la Universidad de Granada. Concretamente se presenta un programa en Mathematica [5] que puede ser usado en asignaturas como Electrodinámica Clásica o Electromagnetismo Avanzado para Ingenieros, y que estudia numéricamente el proceso de radiación en una antena de hilo.**

I. Introduction

In this paper the teaching program "wire rad" is presented. It studies numerically the radiation process in a thin wire antenna excited by a broadband signal. Fundamentally, with the program we can obtain:

- Charge and current distribution on the antenna
- Electric and magnetic field (near and radiated terms separately) in an arbitrary point for every time

Time domain and frequency domain are combined: the source distribution is obtained in frequency domain and then transformed via Discrete Fourier Transform (DFT) to time domain in order to obtain the radiated fields.

The program is fully interactive and it allows us to calculate further antenna parameters such as input impedance, or field magnitudes such as Poynting vector. The user knows what the program is doing due to the clarity of the Mathematica syntax. We have sacrificed a good programing technique in favour of transparency.

Through this paper all the information about the program is given in order to use it in a Classical Electrodynamic class or an Advanced Engineering Electromagnetics class on radiation process.

We solve the Hallen equation to find the frecuency domain current distribution along the wire, The charge density is calculated from the current distribution in time domain using the continuity equation. The radiated field is given by equations derived from Jefimenko's generalization of the Coulomb and Biot-Savart laws.[1]

The Hallen equation is solved numerically using the Method of Moments (MoM) [2], and equivalent approximations are used to evaluate the electric and magnetic fields. This method is widely used in computational Electromagnetism, as in other software such as [4]. The program that is presented in this paper has the following differences:

1. It combines the time domain with frequency domain showing the equivalence and taking advantage of each one.
2. It is written in Mathematica code: the user can follows the calculus and can do extra calculus as well.

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Because the emission of radiation by a localized system may be a difficult problem for the students, we hope with this material they will be able to understand the physics of the problem more easily.

In order to obtain the radiating system response at any frequency, inside the interval in which the approximations performed are valid, we excite the system with a narrow gaussian pulse. In the limit (depending on its duration, a negligible width) it has a broadband spectrum like the δ-Dirac function. Subsequently we can evaluate the system response to any excitation function by convolution.

II. Formulation of the problem and solution via MoM

From the Hallen integral equation [1] we shall find the electromagnetic field sources (surface currents and charges). This equation is easily solved from a numerical point of view because it has no time derivatives of the current intensity in its kernel, and as a consequence it is the only unknown there is. We consider a thin linear antenna of length L which is excited across a small gap on its midpoint. The antenna is assumed to be oriented along the z-axis with its gap on the origin. In order to obtain the current along the antenna, we have to solve a boundary value problem. We assume the antenna is a perfect conducting wire with a small radius compared to both the minimum wavelength of the excitation signal spectrum and the length L. So that the current flowing on the surface has only a longitudinal (z) component, and the fields have azimuthal symmetry. To avoid singularity in the kernel of the integral equations we assume that the current flows in the axis of the antenna, whereas the boundary conditions are applied on the surface [3].

For the fields we use equations (6.55) and (6.56) of Jackson [1] specified for linear density of charge and current, in which the temporal variable appears explicitly. We have to transform the equations in order to write near field and radiation field separately.

Our electromagnetic problem involves the solution of a linear integral equation. We use the point-matching form of the Method of Moments that transforms the integral equation into a linear system of equations.

The intensity current is constant on each of the N_s intervals in which the antenna has been partitioned for its numerical resolution (pulse basic functions). Obviously, the calculation of the linear density charge and the time derivative of the current from the intensity current are made using the interpolation polynomials, in time as in space. So each one of this three variables are approximated by a matrix with N_s rows that correspond to the N_s intervals in which the antenna has been divided, and N_t columns that correspond to the number of time intervals in which the integral equation has been solved. Once the point where we want to calculate the field is fixed, for each spatial interval there is a delayed time that is given approximately by an integer number of time intervals. By the causality principle all the sources are equal to zero for $t \leq 0$ so in order to evaluate the sources at the delayed time it is enough to add a number of zeros to each row equal to the previously mentioned temporal interval integer number. Once these zeros are added we have the sources as are "seen" from the field point, and the calculation of the radiated field implies only a sum by columns multiplied by the factor corresponding due to the integral kernel.

III. Results

As an example we have studied the current distribution on an antenna of 1 meter length, radius $a = 0.00672m$, and feed by a gaussian pulse with parameter $g = 3.0 * 10^9 s^{-1}$, therefore $\lambda_{min} = 0.20, 0.15m$ regarding the effective width pulse up to $1/10$ or $1/100$ from its maximum value. The difference is very small due to the exponential declining of the feeding signal amplitude. Even for the smaller value it is found that λ_{min} is huge compared to the wire radius. The antenna has been divided into 30 segments ($N_s = 15$), so each spatial interval is $0.033m$ length.

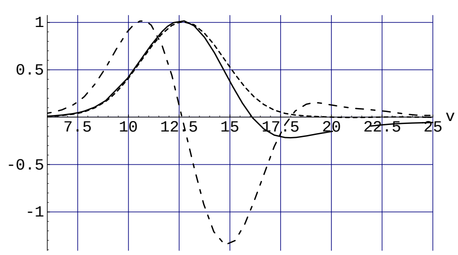

Fig. 1. Early time at the feeding point of the current (solid line), time derivative of the current (dashed line) and charge density (dotted line) in normalized representation

Figure 1 shows the early time response of the current at the feeding point (solid line) together with the time derivative of the intensity current (dashed line) and the charge density (dotted line). It has been normalized to the maximum value for a better representation of the fact that the time derivative of the current has a maximum value some time instants before the charge density and the intensity current. This means that the radiation fields must have a peak value before it occurs at the near fields. Figure 2 represents the z component of the electric field at the point $\vec{x} = (0, 3, 15)\Delta s$ (x and y components are not relevant compared to z) separated in the near field (coulombian, dashed line, inductive, solid line) and the radiation field (dotted line). It can be clearly seen that the last one has a maximum before the other two contributions.

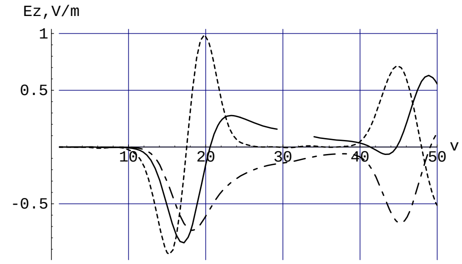

Fig. 2. Ez radiated field at the observation point $\vec{x} = (0, 3, 15)\Delta s$, decomposited on the terms: coulombian (dashed line), inductive (solid line) and radiation field (dotted line)

IV. Conclusions

We think that with this focus that has been given to the analysis of an interesting radiating system, such as the thin wire antenna, it has been possible to provide the student with a tool with which to explore several ideas related to the radiation phenomena, such as the following:

- Setting out the problem of determining, using the boundary condition on the antenna, the sources of the field knowing the excitement and the geometry of the structure.
- Dependence of the different terms of the field with the sources evaluated in delayed time.
- The understanding of the equivalence between the time domain and the frequency domain.
- Introduction to the use of numerical methods.

At the same time solution to a practical problem is demonstrated whilst comprehension of the concepts is achieved.

A concise and clear algorithm has been developed using MATHEMATICA [5]. It allows the user to easily change any parameter that defines the problem and see the results (charge and intensity in frequency domain and time domain, near and far electric field, magnetic field, etc.) on the screen. The solution presented in this paper, the thin wire antenna, is a very good thought provoking problem for the students.

Acknowledgment

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References

[1] J.D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1999, 3th ed.
[2] R.F. Harrington, *Field Computation By Moment Methods*, Macmillan, New York, 1968.
[3] C.A. Balanis, *Advanced Engineering Electromagnetics*, Wiley, New York 1989.
[4] G.J. Burke, and A.J. Poggio " Numerical electromagnetics code (NEC)-method of moments," Technical Document 116, Naval Ocean Systems Center, San Diego, Calif., January 1981
[5] Mathematica is a product of Wolfram Research.[Online] Available http://www.wolfram.com